Alternate Bullet Parameterization to Ballistic Coefficient

Allen Dominek

Abstract

The drag of a bullet is an important parameter. Historically, a bullet's drag force has been characterized with a velocity squared dependency which leads to a complex coefficient of drag that is velocity dependent. To classify the drag of a bullet with one number, a ballistic coefficient was created which is dependent upon a "reference" shape. Shown here, if the drag force is characterized with the velocity raised to 1.5 power, the coefficient of drag becomes velocity independent. The coefficient of drag then naturally becomes an ideal parameter to characterize the bullet's drag in the supersonic range.

Keywords: ballistic coefficient, drag, supersonic

1. Introduction

Ballistic coefficients (BC) are used to characterize a bullet's performance. There are many references that discuss BCs [1] -[6]. The characterization attempts to address the bullet's terminal effect by accounting for its sectional density since the BC units is lbs/in^2. The other parameter of concern is the loss of velocity that the bullet has during flight due to its drag. This is provided by a dimensionless shape factor. The final characterization results in a BC that relates the bullet performance to a reference standard shape. As bullet technology has changed, the reference shapes have varied resulting in BC's that are G1 or G7 based, for example.

There is an alternate parameterization to better characterize a bullet's performance. This characterization focuses upon the drag a bullet has over the whole supersonic range (Mach > 1) with one number, that has no reference to a standard shape. Such a metric is very desirable.

2. Coefficient of Drag

The velocity of a bullet continuously slows down once it leaves the muzzle due to aerodynamic drag. The drag force is analytically represented by

$$F_d = -\frac{1}{2}\rho C_d A v^2 = m_b a_d \tag{1}$$

The parameters in this representation are the bullet's mass, m_b, its velocity, v, its cross-sectional area, A, ρ is the mass density of air and a coefficient drag, C_d. It is important to realize that this force drag is based upon an incompressible fluid flow (the density of the fluid is constant). However at supersonic velocities, the fluid is compressed, generating a shock wave when the Mach velocity is greater than 1.

Rearranging Eq. 1 for the coefficient of drag, C_d, results in

$$C_d = -m_b a_d * 2/(\rho A v^2). \tag{2}$$

The only quantity not readily available in Eq. 2 is the drag deceleration factor, a_d. The deceleration factor can be obtained by taking the temporal derivative of the velocity profiles such as the ones shown in Figure 1. These velocity profiles were obtained with an appropriate ballistic calculator. One calculator is provided by Applied Ballistics [7] since some of their velocity-range-ToF (time of flight) data is based upon measured data.

Figure 1 illustrates a number of velocity vs range data for a number of Hornady bullets. For completeness, this data calculated with a pressure of 29.92 inHg and a relative humidity of 65%. The bullets examined were Interlock BTSP 139 grn, ELD-X 150 grn, ELD-M 162 grain and ELD-X 175 grn each with a 7 mm caliber. The curves identified as "cus" (custom) are based upon measured data.

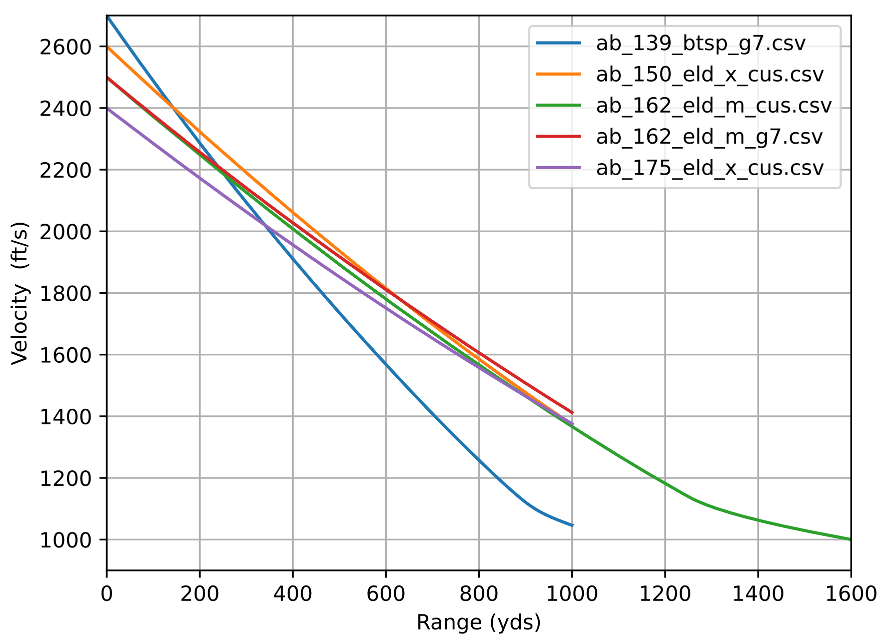

Figure 1: The range dependency of velocity for several bullets.

The temporal derivative can be approximated with this expression

$$a_d \approx \frac{v(d_2) - v(d_1)}{\text{ToF}(d_2) - \text{ToF}(d_1)} \tag{3}$$

where d_1 and d_2 are the range distances and ToF (time of flight) provided by the ballistic calculator. The resulting deceleration curves have a slight ripple due to the first order derivative approximation. This calculator has proven to be accurate for Hornady's ELD-X 162 grain bullet out to 1000 yards. Note that not all ballistic calculators maybe as accurate. The resulting coefficients of drag for these bullets are illustrated in Figure 2 ($\rho = .0023133$ slugs/ft).

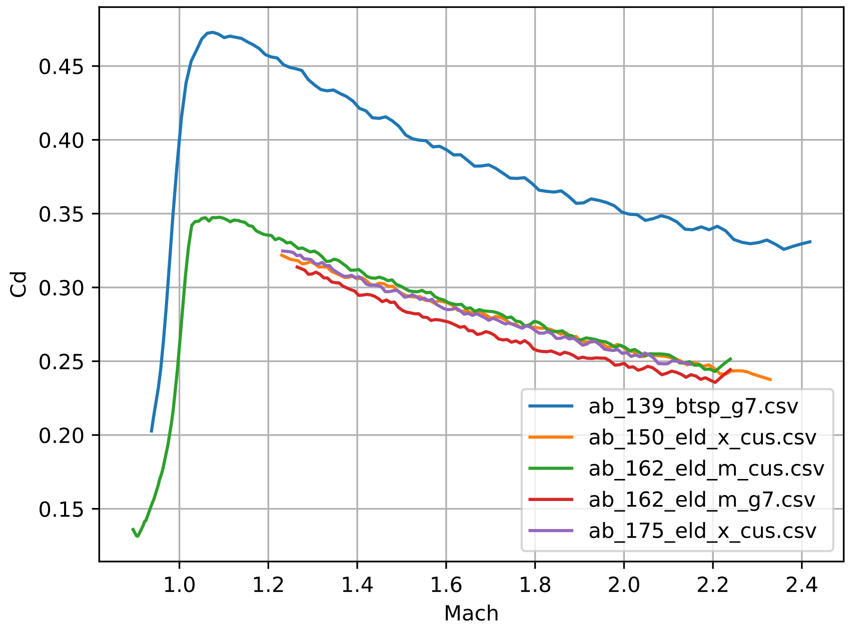

Figure 2: Coefficient of drag for several bullets.

It is interesting to view the numerical values for the drag forces, F_d, and decelerations, a_d, for these bullets as illustrated in Figures 3 and 4.

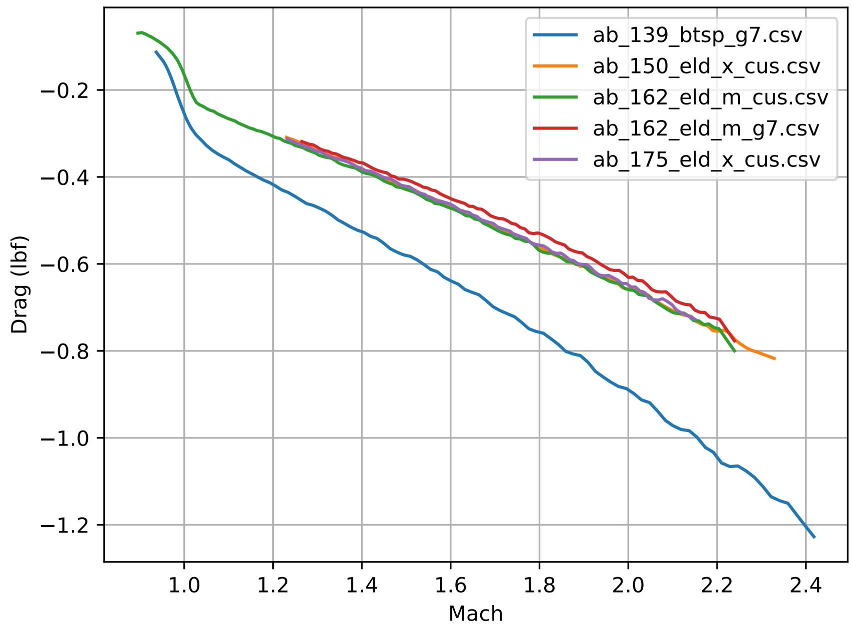

Figure 3: Illustration for the drag force for several bullets.

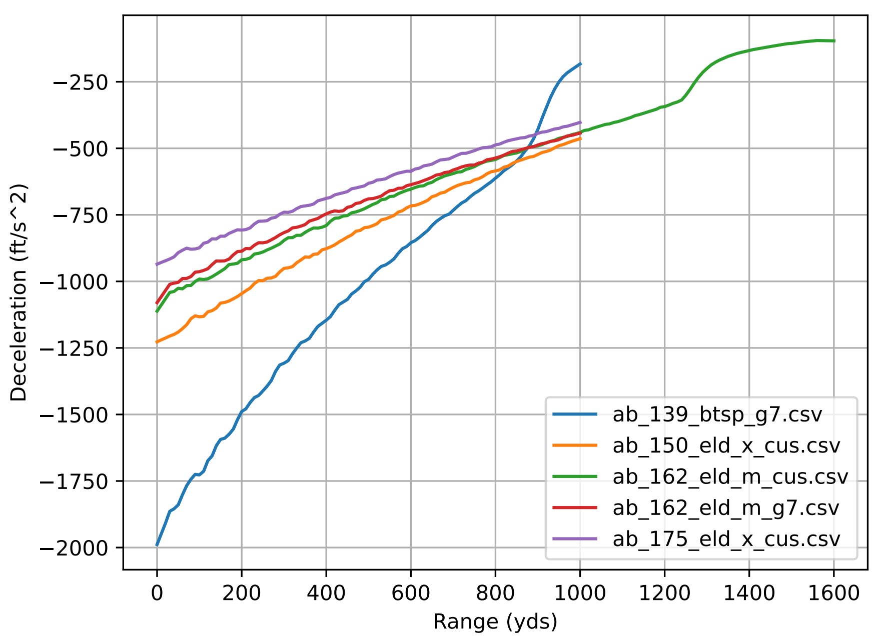

Figure 4: Illustration of the deceleration for several bullets.

It is interesting to note that C_d has a smoothly varying velocity dependency for velocities greater than the speed of sound. This suggests that the drag force may not be a function of velocity squared for bullets with velocities in the supersonic range (Re>1000). The use of a squared dependency is a common practice and is deeply ingrained. When a bullet has a supersonic velocity, the gas around the bullet is compressed. Hence a force representation based upon a based upon an incompressible flow should not be used in this case.

Generally, for a smooth function, drag for example, could be represented by a power series such as

$$F(v) = \sum_{i=0}^{N} a_i v^n. \tag{4}$$

Currently, Eq. 2 has the velocity dependency represented with only one term for an integer power of 2. Note, there is nothing that requires the exponent to be an integer either. It would be ideal if the coefficient of drag was a constant when the bullet has a supersonic velocity. A numerical optimization was performed for the reference bullets using v^x, with Applied Ballistics data, as the velocity dependency to find if a single value of x could be found that would make the coefficient of drag constant. It was found that a value of 1.5 would make the coefficient of drag constant for all five cases using a Nelder-Mead optimization technique. Using an exponent value of 1.5 with a single term as shown in the following expression

$$C_{d1} = -m_b a_d * 2/(\rho A v^{1.5}) \tag{5}$$

which results in the drag coefficients illustrated in Figure 5.

Note now just one number can be used to characterize the performance of a bullet. The coefficient C_{d1} has dimensions of $\sqrt{(ft/s)}$ while C_d is dimensionless which is really not a concern if a coefficient is dimensionless or not. Numerically, these coefficients are given in Table 1.

Table 1: Alternate Drag Coefficient

Bullet	Weight	C_{d1}
BTSP	139	16.675
ELD-X	150	12.157
ELD-M	162	12.283
ELD-X	162	11.704
ELD-X	175	12.112

Figure 6 illustrates a comparison for the Hornady reported BC for these bullets and the alternate coefficient of drag, C_{d1}.

3. Conclusion

Representing the velocity dependency for the drag force with an exponent of 1.5 instead of 2 results in a constant coefficient of drag, a very desirable

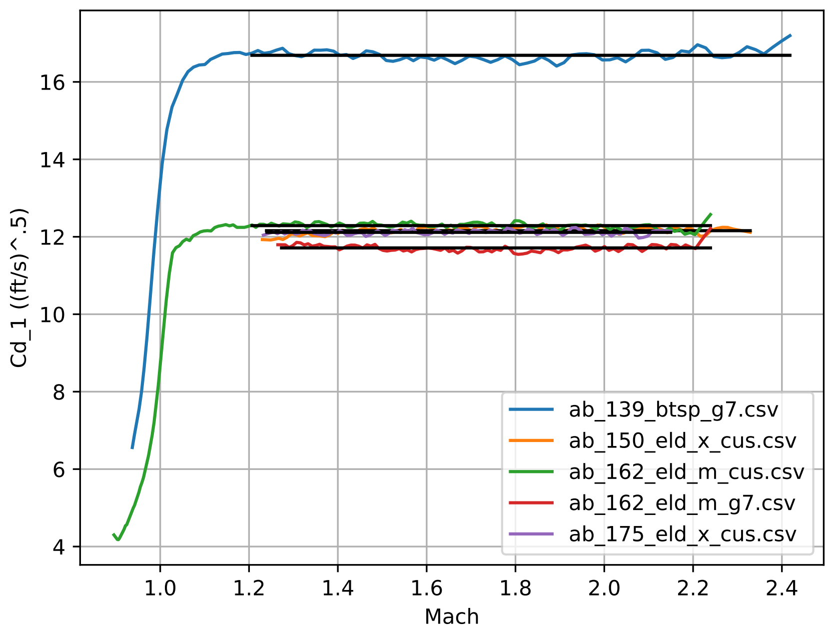

Figure 5: Coefficient of drag based upon $v^{1.5}$.

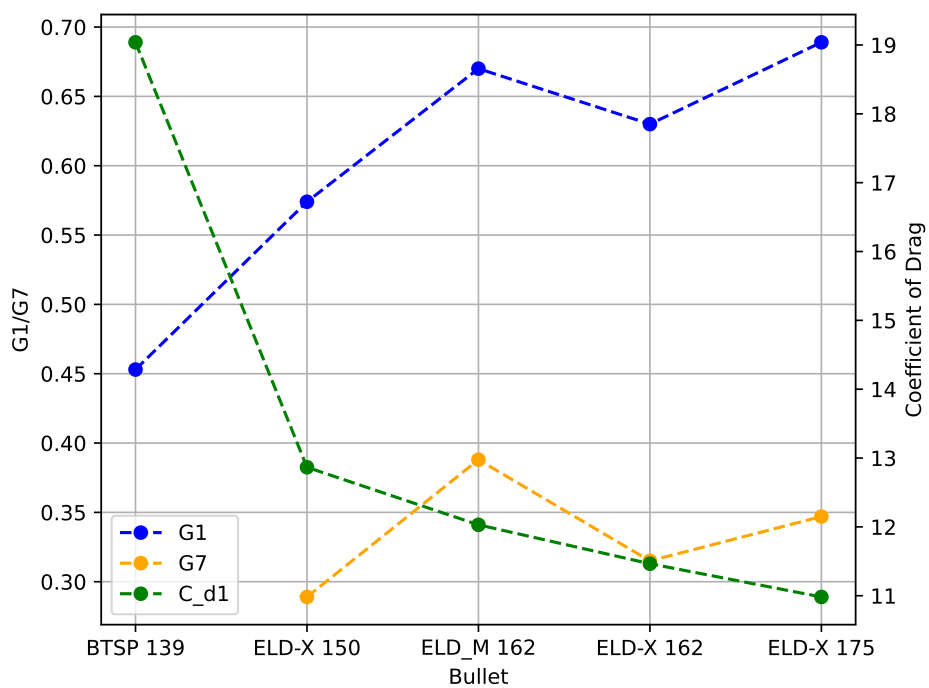

Figure 6: Comparison of the traditional BC and an alternate coefficient of drag for these bullets.

parameterization. The need to use a BC to quantify a bullet's performance based upon a relative shape is not very desirable. It would be desirable to use this alternate coefficient as a figure of merit for bullet performance since one number, which has physical meaning, can represent the performance over the supersonic range of interest.

Acknowledgements

The author would like to thank Sean Gilmore for his interest in this topic and reviewing the manuscript.

References

[1] The Ballistic Coefficient, https://www.sierrabullets.com/exterior-ballistics/2-0-the-ballistic-coefficient/https://www.sierrabullets.com/exterior-ballistics/2-0-the-ballistic-coefficient/, Sierra Bullets.

[2] #32 Ballistic Coefficient, https://thescienceofaccuracy.com/podcast/32-bc-and-how-to-use-it/https://thescienceofaccuracy.com/podcast/32-bc-and-how-to-use-it/, Applied Ballistics.

[3] Ballistic coefficient, https://en.wikipedia.org/wiki/Ballistic_coefficienthttps://en.wikipedia.org/wiki/Ballistic_coefficient.

[4] Dave Emary, "A Technical Discussion of the ELD-X and ELD-Match Bullets with Heat Shield Tip", https://static.hornady.media/site/hornady/files/resources/ELD-X_ELD-Match_Technical_Details.pdfhttps://static.hornady.media/site/hornady/files/resources/ELD-X_ELD-Match_Technical_Details.pdf, Oct. 2015, Hornady Manufacturing.

[5] Gan, Chenjie,"Computational Fluid Dynamics (CFD) of Drag Force for Bullet's Shape Design", Journal of Physics: Conference Series, 2021, 1888,1, https://www.scopus.com/inward/record.uri?eid=2-s2.0-85105498550&doi=10.1088%2F1742-6596%2F1888%2F1%2F012016&partnerID=40&md5=36fe6fb6d5c057a56e0d61604542ef94.

[6] Maxa, J., Šabacká, P., Bayer, R., Binar, T., Bača, P., Švecová, J.,Talár, J., Vlkovský, M., Dobšáková, L.,"The Tuning of a CFD Model for External Ballistics, Followed by Analyses of the Principal Influences on the Drag Coefficient of the .223 Rem Caliber", Technologies 2025, 13, 5, https://doi.org/10.3390/technologies13050190.

[7] Applied Ballistics Mobile App, Applied Ballistics.